

Conductance of a Finite Quantum Wire Connected to Reservoirs

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Abstract

We study a finite quantum wire connected to external leads, and show that the conductance of the system significantly depends upon the length of the quantum wire and the position of the impurity in it. For a very long quantum wire and the impurity far away from its two ends, the conductance has the same behavior as that for an infinity quantum wire above some very little energy scale. However, for a very short quantum wire, the conductance is independent of the electron-electron interactions in it and closing to $e^2/(2\pi\hbar)$ in a higher temperature range. While, in a lower temperature range, the conductance shows the same property as that for an infinity quantum wire.

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Recently, considerable efforts have been directed towards the study of the transport property of one-dimensional(1D) Tomonaga-Luttinger(TL) liquids [1–20]. For an infinity impurity-free quantum wire, the conductance is believed to be $ge^2/(2\pi\hbar)$ per spin orientation, where g is a dimensionless coupling strength parameter of the conduction electrons. For non-interacting electrons, $g = 1$. For repulsive interaction of the conduction electrons, $g < 1$, and the conductance is reduced. However, for a finite impurity-free quantum wire connected leads (reservoirs) which are characterized by Fermi liquid, the conductance is believed to be $e^2/(2\pi\hbar)$ which is independent of the interactions in the quantum wire [16–20], this surprising result derives from the boundary conditions between the TL liquid in the wire and the Fermi liquid in the leads at the ends of the wire. A recent experiment on a longer *GaAs* high-mobility quantum wires [10] shows that for a higher temperature, the conductance is very close to $e^2/(2\pi\hbar)$, but for a lower temperature, the conductance has a power-law temperature dependence behavior, which is believed to be induced by the impurity scattering in the quantum wires. Therefore, this experiment really reveals the physical property of a finite quantum wire with impurity scattering. While, it is well-known that the backward scattering of the conduction electrons induced by the impurity is relevant in terminology of renormalization group, the perturbation methods may fail for treating this kind of system, some results obtained by perturbation methods are not reliable. In Ref. [15], by using bosonization method and unitary transformation, we can exactly treat the backward scattering of the conduction electrons and clearly show that the backward scattering significantly changes the correlation exponents of the conduction electrons. It is very difficult to obtain these correlation exponents by the perturbation methods. In present Letter, encouraged by the exact solution of single quantum impurity scattering in TL-liquid, we study the conductance of a finite quantum wire connected to reservoirs. In contrast with an infinite quantum wire, it shows some different behaviors. The conductance drastically depends upon the length of the quantum wire and the position of the impurity. For a very long quantum wire, generally, it shows the same behavior as that for an infinity quantum wire above some very little energy scale. For a very short quantum wire, the conductance is independent of

the electron-electron interactions in the quantum wire in a higher temperature range, while its temperature dependence has the same form as that for the infinity quantum wire in a lower temperature range.

The Hamiltonian describing the 1D TL-liquid is generally given by

$$H_0 = -i\hbar v_F \int_{-a}^{L-a} dx [\psi_R^\dagger(x) \partial_x \psi_R(x) - \psi_L^\dagger(x) \partial_x \psi_L(x)] \quad (1)$$

$$H_I = \frac{V}{2} \int_{-a}^{L-a} dx (\rho_R(x) + \rho_L(x))^2 \quad (2)$$

$$H_{im} = V_{2k_F} [\psi_R^\dagger(0) \psi_L(0) + \psi_L^\dagger(0) \psi_R(0)] \quad (3)$$

where $\psi_R(x)(\psi_R^\dagger(x))$ is the field operator of fermions that propagate to the right with wave vectors $\sim +k_F$, $\psi_L(x)(\psi_L^\dagger(x))$ is the field operator of left propagating fermions with wave vectors $\sim -k_F$; $\rho_{R(L)}(x) = \psi_{R(L)}^\dagger(x) \psi_{R(L)}(x)$ are the electron density operators; the spectrum of the electrons is linearized near the Fermi points and v_F is the Fermi velocity; V describes density-density interaction with momentum transferring much smaller than k_F . $V_{2k_F} = V(k = 2k_F)$ is the backward scattering potential of an impurity residing at $x = 0$ on the conduction electrons, for simplicity, we have omitted the forward scattering potential because it has less influence on the conductance. L is the length of the quantum wire, and $0 < a \leq L/2$. For simplicity, the reservoirs are assumed to be described by free electron systems.

In the previous bosonization treatment of the Hamiltonians (1), (2) and (3), one directly substitutes the bosonic representation of the fermion fields $\psi_{R(L)}(x)$ into Equ.(3) and obtains a non-linear term which make the system be more difficult treated. To more effectively study the physical property of the system described by the Hamiltonians (1),(2) and (3), we choose other new fermionic field operators

$$\psi_1(x) = \frac{1}{\sqrt{2}}(\psi_R(x) + \psi_L(-x)), \quad \psi_2(x) = \frac{1}{\sqrt{2}}(\psi_R(x) - \psi_L(-x)) \quad (4)$$

It is easy to check that the operators $\psi_{1(2)}(x)$ satisfy the standard anticommutation relations. In terms of these new fermion fields $\psi_{1(2)}(x)$, the cross term of the fermion fields $\psi_{R(L)}(x)$

in (3) can be written in a very simple form which can be cancelled by a simple unitary transformation. However, the Hamiltonian (2) becomes complex. Taking usual bosonization procedure [1,21,22] for $\psi_{1(2)}(x)$, the Hamiltonians (1), (2) and (3) can be written as

$$H_0 = \frac{\hbar v_F}{4\pi} \int_{-a}^{L-a} dx [(\partial_x \Phi_1(x))^2 + (\partial_x \Phi_2(x))^2] \quad (5)$$

$$\begin{aligned} H_I = & \frac{V}{4} \int_{-a}^{L-a} dx \{ [\rho_1(x) + \rho_2(x)]^2 + [\rho_1(x) + \rho_2(x)][\rho_1(-x) + \rho_2(-x)] \\ & + [\psi_1^+(x)\psi_2(x) + \psi_2^+(x)\psi_1(x)]^2 \\ & - [\psi_1^+(x)\psi_2(x) + \psi_2^+(x)\psi_1(x)][\psi_1^+(-x)\psi_2(-x) + \psi_2^+(-x)\psi_1(-x)] \} \end{aligned} \quad (6)$$

$$H_{im} = \frac{\hbar v_F \delta}{2\pi} (\partial_x \Phi_1(x) - \partial_x \Phi_2(x))|_{x=0} \quad (7)$$

where $\delta = \arctan(V_{2k_F}/(\hbar v_F))$ is a phase shift induced by the backward scattering potential V_{2k_F} . This replace of V_{2k_F} by δ can be judged by the usual Born-approximation method and the solution of the X-ray absorption in usual metals [23]. For simplicity, we write out the last two terms in (6) by the fermion fields $\psi_{1(2)}(x)$. If we use the boson fields $\Phi_{1(2)}$ to write out these two terms, they become very complex *cosine* forms. However, no matter which description we use, the final result is same. The bosonization representation of the fermion fields $\psi_{1(2)}$ can be written as [1,21,22]

$$\psi_{1(2)}(x) = \left(\frac{D}{2\pi\hbar v_F}\right)^{1/2} \exp\{-i\Phi_{1(2)}(x)\} \quad (8)$$

where D is the band width of the conduction electrons in the quantum wire, $\rho_{1(2)}(x) = \psi_{1(2)}^+(x)\psi_{1(2)}(x)$ are the density operators of the fermion fields $\psi_{1(2)}(x)$, and have relations with the boson fields $\Phi_{1(2)}(x)$: $\partial_x \Phi_{1(2)}(x) = 2\pi\rho_{1(2)}(x)$. In terms of the new boson and fermion fields $\Phi_{1(2)}(x)$ and $\psi_{1(2)}(x)$, the Hamiltonian (6) becomes complex, but the Hamiltonian (7) becomes very simple. Generally, due to the simple form of the Hamiltonian (7) which is proportional to the density of the fermion fields $\psi_{1(2)}(x)$ at the impurity site $x = 0$, using an unitary transformation we can eliminate it, so that the problem is simplified as that we only need to treat a Hamiltonian similar to (5) and (6). However, the backward scattering

interaction drastically influences the behavior of the conduction electrons through changing interactions among them. Therefore, we cannot simply eliminate the backward scattering term by an unitary transformation and meanwhile leave the Hamiltonians (5) and (6) intact.

To cancel the δ -term in (7) and simplify the system, we adopt the following steps [15]:

1). Taking the unitary transformation

$$U = \exp\left\{i\frac{\delta}{2\pi}(\Phi_1(0) - \Phi_2(0))\right\} \quad (9)$$

we have the following relations

$$\begin{aligned} U^+(H_0 + H_{im})U &= H_0 \\ U^+H_IU &= \frac{V}{4} \int_{-a}^{L-a} dx \{ [\rho_1(x) + \rho_2(x)]^2 + [\rho_1(x) + \rho_2(x)][\rho_1(-x) + \rho_2(-x)] \\ &\quad + [e^{-i\delta \operatorname{sgn}(x)}\psi_1^+(x)\psi_2(x) + e^{i\delta \operatorname{sgn}(x)}\psi_2^+(x)\psi_1(x)]^2 \\ &\quad - [e^{-i\delta \operatorname{sgn}(x)}\psi_1^+(x)\psi_2(x) + e^{i\delta \operatorname{sgn}(x)}\psi_2^+(x)\psi_1(x)] \\ &\quad \cdot [e^{i\delta \operatorname{sgn}(x)}\psi_1^+(-x)\psi_2(-x) + e^{-i\delta \operatorname{sgn}(x)}\psi_2^+(-x)\psi_1(-x)] \} \end{aligned}$$

2). Performing the gauge transformations

$$\psi_{1(2)}(x) = \bar{\psi}_{1(2)}(x)e^{i\theta_{1(2)}}, \quad \theta_1 - \theta_2 = \delta \quad (10)$$

we have the relations

$$\begin{aligned} U^+H_IU &= \bar{H}_I^{(1)} + \bar{H}_I^{(2)} \\ \bar{H}_I^{(1)} &= \frac{V}{4} \int_{-a}^{L-a} dx \{ [\rho_1(x) + \rho_2(x)]^2 + [\rho_1(x) + \rho_2(x)][\rho_1(-x) + \rho_2(-x)] \\ &\quad + [\bar{\psi}_1^+(x)\bar{\psi}_2(x) + \bar{\psi}_2^+(x)\bar{\psi}_1(x)]^2 \\ &\quad - \cos(2\delta)[\bar{\psi}_1^+(x)\bar{\psi}_2(x) + \bar{\psi}_2^+(x)\bar{\psi}_1(x)][\bar{\psi}_1^+(-x)\bar{\psi}_2(-x) + \bar{\psi}_2^+(-x)\bar{\psi}_1(-x)] \} \\ \bar{H}_I^{(2)} &= \frac{V}{4} \int_0^{L-a} dx \left\{ \frac{\cos(4\delta) - 1}{2} [\bar{\psi}_1^+(x)\bar{\psi}_2(x) + \bar{\psi}_2^+(x)\bar{\psi}_1(x)]^2 \right. \\ &\quad + [\bar{\psi}_1^+(x)\bar{\psi}_2(x) - \bar{\psi}_2^+(x)\bar{\psi}_1(x)]^2 \} \\ &\quad + i \sin(2\delta) \int_{-a}^{L-a} dx [\bar{\psi}_1^+(x)\bar{\psi}_2(x) - \bar{\psi}_2^+(x)\bar{\psi}_1(x)] \\ &\quad \cdot [\bar{\psi}_1^+(-x)\bar{\psi}_2(-x) + \bar{\psi}_2^+(-x)\bar{\psi}_1(-x)] \end{aligned}$$

3). Re-defining the left- and right-moving electron fermions

$$\begin{aligned}\bar{\psi}_R(x) &= \frac{1}{\sqrt{2}}[\bar{\psi}_1(x) + \bar{\psi}_2(x)], \quad \bar{\psi}_L(-x) = \frac{1}{\sqrt{2}}[\bar{\psi}_1(x) - \bar{\psi}_2(x)] \\ \bar{\psi}_{R(L)}(x) &= \left(\frac{D}{2\pi\hbar v_F}\right)^{1/2} \exp\{-i\bar{\Phi}_{R(L)}(x)\}, \quad \partial_x \bar{\Phi}_{R(L)}(x) = \pm 2\pi\bar{\rho}_{R(L)}(x)\end{aligned}\quad (11)$$

where $\bar{\rho}_{R(L)}(x) = \bar{\psi}_{R(L)}^\dagger(x)\bar{\psi}_{R(L)}(x)$ are the density operators of the electron fields $\bar{\psi}_{R(L)}(x)$, the Hamiltonians $\bar{H}_I^{(1)}$ and $\bar{H}_I^{(2)}$ can be rewritten as

$$\begin{aligned}\bar{H}_I^{(1)} &= \frac{V}{2} \int_{-a}^{L-a} dx \{[\bar{\rho}_R(x) + \bar{\rho}_L(x)]^2 \\ &\quad + \frac{1 - \cos(2\delta)}{2} [\bar{\rho}_R(x) - \bar{\rho}_L(-x)][\bar{\rho}_R(-x) - \bar{\rho}_L(x)]\} \\ \bar{H}_I^{(2)} &= i \frac{V \sin(2\delta)}{4} \int_{-a}^{L-a} dx [\bar{\rho}_R(-x) - \bar{\rho}_L(x)][\bar{\psi}_L^\dagger(-x)\bar{\psi}_R(x) - \bar{\psi}_R^\dagger(x)\bar{\psi}_L(-x)] \\ &\quad + \frac{V}{8}(1 - \cos(4\delta)) \int_0^{L-a} dx [\bar{\rho}_R(-x)\bar{\rho}_L(x) - \bar{\rho}_R(x)\bar{\rho}_L(-x)]\end{aligned}$$

It is worth notice that the Hamiltonian $\bar{H}_I^{(2)}$ only contributes high order corrections because the first term has the conformal dimension $\Delta \geq 2$ and the last term has the conformal dimension $\Delta = 2$, and at the weak ($\delta \sim 0$) and strong ($\delta \sim \pm\pi/2$) coupling limit, they all tend to zero. Therefore, for simplicity we can neglect it.

4). Defining a set of new boson fields

$$\begin{aligned}\Theta_+(x) &= \left(\frac{G(\delta)}{\cosh(\chi_1 - \chi_2)}\right)^{1/2} [\cosh(\chi_1)\Phi_+(x) - \sinh(\chi_1)\Phi_+(-x)] \\ \Theta_-(x) &= \left(\frac{1}{G(\delta)\cosh(\chi_1 - \chi_2)}\right)^{1/2} [\cosh(\chi_2)\Phi_-(x) - \sinh(\chi_2)\Phi_-(-x)]\end{aligned}\quad (12)$$

where $G(\delta) = [(1-\gamma)(1-\gamma\cos(2\delta))]^{1/4}/[(1+\gamma)(1+\gamma\cos(2\delta))]^{1/4}$, $\tan(2\chi_1) = \beta\gamma/(1-\alpha\gamma)$, $\tan(2\chi_2) = \beta\gamma/(1+\alpha\gamma)$, $\alpha = (1+\cos(2\delta))/2$, $\beta = (1-\cos(2\delta))/2$, $\Phi_\pm(x) = [\bar{\Phi}_R(x) \pm \bar{\Phi}_L(x)]/\sqrt{2}$.

The total Hamiltonian of the system can be simplified as a very simple form

$$\bar{H} = \frac{\hbar\bar{v}_F}{4\pi g} \int_{-a}^{L-a} dx [(\partial_x \Theta_+(x))^2 + (\partial_x \Theta_-(x))^2] \quad (13)$$

where $\bar{v}_F = v_F \cosh(\chi_1 - \chi_2) \left(\frac{1 - (\gamma \cos(2\delta))^2}{1 - \gamma^2} \right)^{1/4}$, $g = \left(\frac{1 - \gamma}{1 + \gamma} \right)^{1/2}$ is a dimensionless coupling strength parameter, where $\gamma = V/(2\pi\hbar v_F + V)$. Here we have omitted some higher order terms [15] which only give less important high order correction to the conductance. However, the dual boson fields $\Theta_{\pm}(x)$ satisfy the following commutation relations

$$\begin{aligned} [\partial_x \Theta_+(x), \Theta_-(y)] &= i2\pi\delta(x - y) + i2\pi\delta(x + y) \tan(\chi_1 - \chi_2) \\ [\partial_x \Theta_-(x), \Theta_+(y)] &= i2\pi\delta(x - y) + i2\pi\delta(x + y) \tan(\chi_1 - \chi_2) \end{aligned} \quad (14)$$

which have an anomaly term $i2\pi\delta(x + y) \tan(\chi_1 - \chi_2)$. This term is zero at the weak $\delta \sim 0$ and strong $\delta \sim \pm\pi/2$ coupling limits. According to these commutation relations of the dual boson fields $\Theta_{\pm}(x)$, for example, we can define the conjugate momentum field $P_-(x)$ of the boson field $\Theta_-(x)$ as

$$-\frac{1}{2\pi} \partial_x \Theta_+(x) = P_-(x) + \tan(\chi_1 - \chi_2) P_-(-x), \quad [P_-(x), \Theta_-(y)] = -i\delta(x - y) \quad (15)$$

and then we can exactly solve the Hamiltonian (13). Therefore, we can calculate the conductance for any backward scattering potential.

For simplicity, we first consider a special case: $\delta = 0$, which corresponds to the weak backward scattering limits. In this case, the propagators of the boson field $\Theta_-(x)$ satisfy the following equation

$$\left\{ -\partial_x \left(\frac{v}{g} \partial_x \right) + \frac{\tilde{\omega}^2}{vg} \right\} G_{\tilde{\omega}}^-(x, x') = \delta(x - x') \quad (16)$$

where $G_{\tilde{\omega}}^-(x, x') = \frac{1}{2\pi} \int_0^{1/(k_B T)} d\tau < T_{\tau} \Theta_-(x, \tau) \Theta_-(x', 0) > e^{i\tilde{\omega}\tau}$, where T is temperature, $v = v_F/g$. Using the boundary conditions [16] of the propagator $G_{\tilde{\omega}}^-(x, x')$, we can easily obtain the following equation at the impurity site $x = 0$

$$G_{\tilde{\omega}}^-(0, 0) \simeq \frac{K}{2|\tilde{\omega}|} \quad (17)$$

where the dimensionless coupling strength parameter K satisfies the following relations

$$K = \begin{cases} g, & |\tilde{\omega}| \gg v/a \\ \frac{2g}{g+1}, & v/L \ll |\tilde{\omega}| \ll v/a \\ 1, & |\tilde{\omega}| \ll v/L \end{cases} \quad (18)$$

However, for a general phase shift δ , we can obtain the following expression of the propagator $G_{\tilde{\omega}}^-(x, x')$ at the impurity site $x = 0$

$$G_{\tilde{\omega}}^-(0, 0) \simeq \frac{G(\delta)\bar{K}}{2\Gamma|\tilde{\omega}|} \quad (19)$$

where the dimensionless coupling strength parameter \bar{K} satisfies the following relations

$$\bar{K} = \begin{cases} 1, & |\tilde{\omega}| \gg v'/a \\ \frac{2\Gamma}{\Gamma + G(\delta)}, & v'/L \ll |\tilde{\omega}| \ll v'/a \\ \frac{\Gamma}{G(\delta)}, & |\tilde{\omega}| \ll v'/L \end{cases} \quad (20)$$

where $\Gamma = [1 + \tan(\chi_1 - \chi_2)]/[1 + \tan^2(\chi_1 - \chi_2)]$, $v' = \bar{v}_F/g$.

Now we calculate the conductance of the system. To this end, we first define a charge density operator $Q(x)$, and then use the continuous equation to obtain the current density operator $J(x)$. The charge density operator $Q(x)$ is equal to $e(\rho_R(x) + \rho_L(x))$. Under the unitary and gauge transformations (9) and (10), it can be written as

$$U^+ \rho_R(x) U = \begin{cases} \bar{\rho}_R(x) - \beta(\bar{\rho}_R(x) - \bar{\rho}_L(-x)) + \\ -i \sin(2\delta)(\bar{\psi}_L^+(-x)\bar{\psi}_R(x) - \bar{\psi}_R^+(x)\bar{\psi}_L(-x)), & x > 0 \\ \bar{\rho}_R(x), & x < 0 \end{cases} \quad (21)$$

$$U^+ \rho_L(x) U = \begin{cases} \bar{\rho}_L(x), & x > 0 \\ \bar{\rho}_L(x) + \beta(\bar{\rho}_R(-x) - \bar{\rho}_L(x)) + \\ -i \sin(2\delta)(\bar{\psi}_L^+(-x)\bar{\psi}_R(x) - \bar{\psi}_R^+(x)\bar{\psi}_L(-x)), & x < 0 \end{cases} \quad (22)$$

It is worth noting that if the phase shift δ takes the values $\pm\pi/2$, the electrons are completely reflected at the impurity site $x = 0$, and we have the relations: $U^+ \rho_R(x) U = \bar{\rho}_L(-x)$ for $x > 0$, and $U^+ \rho_L(x) U = \bar{\rho}_R(-x)$ for $x < 0$. Therefore, the phase shift $\delta^c = \pm\pi/2$ correspond to the strong coupling critical points of the system. However, there exists a gauge symmetry in the system, if we take $\theta_1 - \theta_2 = -\delta$, we can have the relations at the strong coupling critical points: $U^+ \rho_R(x) U = \bar{\rho}_L(-x)$ for $x < 0$, and $U^+ \rho_L(x) U = \bar{\rho}_R(-x)$ for $x > 0$. Accordingly, for $x > 0$, we can obtain the following current density operator

$$\begin{aligned}
J(x) = & \frac{e}{2\pi} \left(\frac{\cosh(\chi_1 - \chi_2)}{2} \right)^{1/2} \left\{ -\beta \left(\frac{g_1(\delta)}{G(\delta)} \right)^{1/2} [\partial_\tau \Theta_+(x) - \partial_\tau \Theta_+(-x)] \right. \\
& + (g_2(\delta)G(\delta))^{1/2} [\partial_\tau \Theta_-(x) - \partial_\tau \Theta_-(-x)] \\
& + \left(\frac{G(\delta)}{g_2(\delta)} \right)^{1/2} [\partial_\tau \Theta_-(x) + \partial_\tau \Theta_-(-x)] \left. \right\} \\
& - i \sin(2\delta) \int_0^x dy \partial_\tau [\bar{\psi}_L^+(-y) \bar{\psi}_R(y) - \bar{\psi}_R^+(y) \bar{\psi}_L(-y)]
\end{aligned} \tag{23}$$

where $g_1(\delta) = (1 - \gamma)^{1/2} / (1 - \gamma \cos(2\delta))^{1/2}$, $g_2(\delta) = (1 + \gamma \cos(2\delta))^{1/2} / (1 + \gamma)^{1/2}$. Using Kubo formula of the conductance, and the expression of the propagators $G_{\tilde{\omega}}^-(x \sim 0, x' \sim 0)$ in (19) and (20), we can obtain the following electric conductance

$$\sigma_{\tilde{\omega}}(x \sim x' \sim 0) = \begin{cases} \frac{e^2}{2\pi\hbar} \frac{\alpha^2 G(\delta) \cosh(\chi_1 - \chi_2)}{g_2(\delta)\Gamma} + A\tilde{\omega}^{2\mu}, & |\tilde{\omega}| \gg v'/a \\ \frac{e^2}{2\pi\hbar} \frac{\alpha^2 \cosh(\chi_1 - \chi_2)}{g_2(\delta)} + B\tilde{\omega}^{2\nu}, & |\tilde{\omega}| \ll v'/L \end{cases} \tag{24}$$

where $\mu = G(\delta) \cosh(\chi_1 - \chi_2) / (g_2(\delta)\Gamma)$, $\nu = \cosh(\chi_1 - \chi_2) / g_2(\delta)$, A and B are constants. This is our central result of present paper. The temperature dependence of the electric conductance $\sigma_{\tilde{\omega}}(x \sim x' \sim 0)$ can be obtained through replacing the frequency $\tilde{\omega}$ by the temperature T . It is necessary to mention that as the frequency $\tilde{\omega}$ and the temperature T tend to zero, $\{\tilde{\omega}, T\} \rightarrow 0$, the phase shift δ takes the values $\pm\pi/2$, and the electric conductance is equal to zero. This behavior can be easily understood by using the renormalization group [5] that because the backward scattering term is relevant, the renormalized backward scattering potential \bar{V}_{2k_F} goes to infinity in the low energy limit, therefore, the phase shift δ induced by the backward scattering potential takes the values $\pm\pi/2$. The electric conductance (24) does not contradict previous work. As $\{\tilde{\omega}, T\} = 0$, the electrons are completely reflected on the impurity site $x = 0$ for the repulsive electron-electron interaction. It is worth notice that the electric conductance significantly depends upon the length of the wire, and the position of the impurity. In generally, for a very long wire, $L \rightarrow \infty$, the electric conductance is in the range of $|\tilde{\omega}| \gg v'/a$. For an impurity-free system, $\delta = 0$, the electric conductance is equal to $ge^2/(2\pi\hbar)$. If there exists impurity scattering, the electric conductance is $\alpha^2 e^2/(2\pi g\hbar)$ in the low energy limit. All these properties are the same as that for an infinite quantum wire. However, for a short quantum wire, in the low energy limit, the electric conductance

falls in the range of $|\tilde{\omega}| \ll v'/L$, for the impurity-free case, the electric conductance is equal to $e^2/(2\pi\hbar)$ which is independent of the electron-electron interactions in the quantum wire [16–20]. As including the impurity scattering, for the weak backward scattering, the electric conductance is still very closing to $e^2/(2\pi\hbar)$. For the strong backward scattering, the electric conductance is the same as that for the infinite quantum wire. However, to consider the temperature dependence of the conductance, we must calculate the tunneling conductance at the impurity site $x = 0$ which derives from the quantum fluctuation of collective excitation modes of the system, because Eq.(24) only gives a higher order temperature dependence.

To calculate the tunneling conductance of the system, we can define the following tunneling current operator at the impurity site $x = 0$ as

$$I_{tunn} = \frac{et_0}{2}(\psi_R^\dagger(0)\psi_L(0) + \psi_L^\dagger(0)\psi_R(0))$$

where t_0 is the tunneling probability amplitude. This definition and the following calculation of the tunneling conductance are meaningful only at the strong coupling region determined by the phase shift δ . In terms of the boson fields $\Theta_\pm(x)$, it can be written as

$$I_{tunn} = \frac{et_0D}{2\pi\hbar v_F} \cos\left[\left(\frac{2G(\delta) \cosh(\chi_1 - \chi_2)}{g_2(\delta)}\right)^{1/2} \Theta_-(0)\right] \quad (25)$$

Therefore, by using Kubo formula of the conductance, we can obtain the following tunneling conductance

$$\bar{\sigma}(\tilde{\omega}) \sim \begin{cases} \tilde{\omega}^{2(\mu-1)}, & |\tilde{\omega}| \gg v'/a \\ \tilde{\omega}^{2(\nu-1)}, & |\tilde{\omega}| \ll v'/L \end{cases} \quad (26)$$

It is worth notice that in the weak coupling fixed point, $\delta = 0$, the exponents μ and ν take the values: $\mu = g$, and $\nu = 1$. While in the strong coupling critical points, $\delta^c = \pm\pi/2$, the exponents μ and ν take the values: $\mu = \nu = 1/g$. However, in the range of $v'/L \ll |\tilde{\omega}| \ll v'/a$, the exponents μ and ν also depend upon the parameters a and L . In generally, it is difficult to compare with the experimental data in Ref. [10]. Based upon Eqs.(24) and (26), we can unambiguously obtain the following conclusions that for a very long quantum wire, $L \rightarrow \infty$, the usual experimental energy falls in the range of $|\tilde{\omega}| \gg v'/L$, if the impurity site is

far away from the ends of the quantum wire, the conductance of the system shows the same behavior as that for an infinite quantum wire. While, for the case of a short quantum wire, the usual experimental energy is in the range of $|\tilde{\omega}| \ll v'/L$, the conductance is independent of the electron-electron interactions in the quantum wire in a higher temperature range (but still leaving the condition $|\tilde{\omega}| \ll v'/L$ intact), and its temperature dependence is the same as that for an infinite quantum wire in the lower temperature range.

In summary, using the bosonization method and the unitary transformation, we have studied the system of a finite quantum wire connected to leads, and shown that in contrast with an infinite quantum wire, it significantly relies upon the length of the wire and the position of the impurity in the wire. For a very short quantum wire, in the higher temperature range, the conductance is independent of the electron-electron interactions in the wire and is very closing to $e^2/(2\pi\hbar)$. However, in the lower temperature range, it is the same as that for an infinite quantum wire.

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